Which are the best maths exhibits shown in mathematics museums around the world?

A survey to the mathematics museums community
edited by Daniel Ramos
IMAGINARY

Version 3.2
Pre-publication release
June 1, 2020

Chapter 1

Foreword

The idea for this survey came up in a meeting at the Mathematisches Forschungsinstitut in Oberwolfach (MFO) in November 2019 between Manjul Bhargava, Fields Medalist and Trustee of the National Museum of Mathematics (MoMath), and IMAGINARY team members Andreas Matt, Bianca Violet and Daniel Ramos.

IMAGINARY conducted a survey between January 13 and January 25, 2020 with most known-existing mathematics museums, with the support of MoMath. The results of the report are shared here with all participating museums. In the forthcoming MATRIX x IMAGINARY conference (`https://matrix.imaginary.org`), originally planned for September 2020 but now delayed due to the COVID-19 pandemic, the report will be presented and made available to the general public through the site `https://www.mathcom.wiki`.

We sent personalized calls for participation to 36 math and science museums that we considered that could provide valuable information. We had a positive response from 15 museums, most of them sending several exhibits that they consider their best. In total we received information about 38 individual exhibits. We selected and curated the answers, having compiled here one exhibit to our choice from each submission.

We are very grateful to all participating museums for their collaboration by sharing public information about their organizations in a manner that can benefit others in creating new museums and spreading the mathematics culture.

Daniel Ramos
Chief Content Officer
IMAGINARY

`daniel.ramos@imaginary.org`

Chapter 2

Museums

On this chapter we compile a few data about each museum, including contact information (not to be released to the public). The museums are ordered alphabetically.

Deutsches Museum

Museum	Deutsches Museum München
Web	`http://www.deutsches-museum.de/en/exhibitions/natural-sciences/mathematics/`
Address	Museumsinsel 1 80538 München (Germany)
Opening year	1999
Exhibition surface	150 m^2
N. of exhibits	40
N. of visitors (yearly)	1 000 000 (whole museum)

The Deutsches Museum in Munich contains a rich mathematics section, designed, curated, created, and maintained by the team and workshops of the Deutsches Museum.

Erlebnisland Mathematik

Museum	Erlebnisland Mathematik
Web	`http://www.erlebnisland-mathematik.de/`
Address	Junghansstraße 1-3 01277 Dresden (Germany)
Opening year	2008
Exhibition surface	1000 m^2
N. of exhibits	100
N. of visitors (yearly)	97 000 (2019)

Located in the Technical Museum of Dresden (Technische Sammlungen Dresden), it occupies one floor of the building. Most of the exhibits are hands-on. They use multimedia, screens and electronics but minimized to the strictly necessary. Exhibits are mostly built in-house, and they sell/license exhibits on demand.

Fermat Science

Museum	Fermat Science
Web	`http://www.fermat-science.com/`
Address	Fermat Science Maison Natale de Pierre Fermat 3 rue Pierre Fermat 82500 Beaumont de Lomagne (France)
Opening year	1996 (association), 2011 (museum)
Exhibition surface	80 m^2
N. of exhibits	40
N. of visitors (yearly)	4 000 students, plus 7 000 visitors

Fermat Science, also known as La Maison de Fermat, is an association and math museum located in the actual house where the 17th century French mathematician Pierre de Fermat was born, in the village of Beaumont de Lomagne (near Toulouse, France). The museum is highly focused on school visits, with 5 mediators full time and school visits every day.

Haus der Mathematik

Museum	Haus der Mathematik, Pädagogische Hochschule Wien
Web	`http://www.hausdermathematik.at/`
Address	Grenzackerstraße 18 1100 Wien (Austria)
Opening year	28 Feb 2003, relocated in 2009
Exhibition surface	160 m^2
N. of exhibits	60
N. of visitors (yearly)	1 200

Il giardino di Archimede

Museum	Il Giardino di Archimede
Web	`http://web.math.unifi.it/archimede/`
Address	Via San Bartolo a Cintoia 19a 50142 Firenze (Italy) tel 055-7879594 , fax 055-7333504 e-mail:`archimede@math.unifi.it`
Opening year	1992 (first exhibition in Pisa), 1999 (Priverno), 2004 (current location in Firenze)
Exhibition surface	1 000 m^2
N. of exhibits	60
N. of visitors (yearly)	13 000

The museum is divided into thematic exhibitions, which have normally existed previously as temporary installations. The different sections are physically in different rooms and they can be visited independently. These exhibitions are: *Beyond the compass*, about the geometry of curves; *A bridge over the Mediterranean*, about Leonardo de Pisa (Fibonacci); *Pythagoras and his theorem*; and *Helping Nature, from Galileo's machines to everyday life*, about basic machines such as levers, pulleys and leaning planes.

Matemateca

Museum	Centro de Difusão e Ensino Matemateca
Web	`http://matemateca.ime.usp.br/`
Address	
Opening year	Oct 2004
Exhibition surface	250 m^2 (temporary ehibition)
N. of exhibits	40 (per exhibition), 70 (full collection)
N. of visitors (yearly)	5 000 per big exhibiton, 500 per small exhibition. Not regular schedule of exhibitions.

The Matemateca is a project associated to the University of São Paulo.

Mathematikum

Museum	Mathematikum
Web	`http://www.mathematikum.de`
Address	Liebigstraße 8 35390 Giessen (Germany)
Opening year	2002
Exhibition surface	1 200 m^2
N. of exhibits	200+ permanent, 250+ temporary
N. of visitors (yearly)	120 000

Mathematikum is one of the first math museums in the world, and it has had an undeniable influence in other math museums, both in Germany as abroad.

It definitely opts for physical objects and an interaction driven from a visitor's point of view, with many big exhibits that involve using the full body, and many mechanical artifacts. There are only 5 of the exhibits which are computer-based or electronic.

All exhibits are designed in-house, and built and repaired by their own workshop. Additionally Mathematikum sells exhibits as part of its business.

MathsWorldUK

Museum	MathsWorldUK
Web	`http://www.mathsworlduk.com/`
Address	-
Opening year	-
Exhibition surface	-
N. of exhibits	20
N. of visitors (yearly)	8 000 (at fairs, etc.)

MathsWorldUK is a project to create a math museum in a major city in the UK. Currently, the team organizes around 8-10 fairs and temporary exhibitions per year.

Meet Math Museum

Museum	Meet Math Museum - Al Quds University
Web	`http://www.mmm.alquds.edu/ar/`
Address	University street / Al Quds University Jerusalem (Palestine)
Opening year	Dec 2007
Exhibition surface	650 m^2
N. of exhibits	45
N. of visitors (yearly)	40 000

Meet Math museum depends on many departments at the Al Quds University (which acts as an incubator). Many professors and staff from the university helps on the development of the ehibits, and on the attention to visitors.

MiMa

Museum	MiMa – Museum für Mineralien und Mathematik
Web	`http://www.mima.museum`
Address	Schulstraße 5 77709 Oberwolfach (Germany)
Opening year	Jan 2010 for math section (minerals museum is older)
Exhibition surface	100 m^2 for math section
N. of exhibits	10
N. of visitors (yearly)	8 000

MiMa museum added a math section due to the neighboring Institute of Mathematical Research of Oberwolfach (MFO). That section opened as a permanent showcase of the successful exhibition IMAGINARY - Through the eyes of mathematics, but it also included other material. Exibits consist on 3 hands-on, a gallery with 24 images, 5 touchscreens with over 35 apps and videos, and a glass case with 39 small sculptures.

MMACA

Museum	Museu de Matemàtiques de Catalunya (MMACA)
Web	`http://www.mmaca.cat/`
Address	Associació mmaca Palau Mercader Carretera Hospitalet, s/n 08940 Cornellà de Llobregat (Spain)
Opening year	Feb 2014
Exhibition surface	300 m^2
N. of exhibits	100+
N. of visitors (yearly)	40 000

Museo de matemáticas de Casbas

Museum	Museo de Matemáticas Monasterio de Casbas
Web	`https://museodematematicas.unizar.es`
Address	Monasterio de Casbas Casbas de Huesca, Huesca (Spain)
Opening year	Jul 2019
Exhibition surface	220 m^2
N. of exhibits	25
N. of visitors (yearly)	100 per day, opens only on summer season

This small and recent math museum is located in a small village in the foot of the Pyrenees mountains, in a 12th century monastery. It opens during the summer season in coordination with school visits and tourist season. It is a joint initiative from University of Zaragoza and the local math teachers association.

Patras Science Center

Museum	Patras Science Centre
Web	https://patrassciencecentre.wordpress.com/
Address	Ag. Paraskevis st. 26504 Platani Patras (Greece)
Opening year	Feb 2009
Exhibition surface	620 m^2
N. of exhibits	90
N. of visitors (yearly)	2 500

Patras Science Center develops its activities in the fields of Informatics, Mathematics and Physics, with the sections "In the spirit of Informatics", "The world of Mathematics" and "The laws of Nature". The public of Patras Science Center are pupils, students and teachers/educators; with schools from across Greece visit the center, and teachers participating in teacher training activities.
https://www.youtube.com/watch?v=EkaWCqUN79Q

Ramanujan Math Park

Museum	Ramanujan Math Park
Web	https://gyanome.org/ramanujan-maths-park/
Address	Agastya Foundation Campus Kuppam (India)
Opening year	Dec 2017
Exhibition surface	5 000 m^2 (outdoors and indoors)
N. of exhibits	50
N. of visitors (yearly)	30 000

Ramanujan Math Park is located in one of the campuses of the Agastya foundation (www.agastya.org), an organization supporting education projects across India. The exhibits are conceived and created by V. S. Sastry, math communicator, and Sujatha Ramdorai, math researcher. Construction is done locally.

Tekniska

Museum	National Museum of Science and Technology
Web	`https://www.tekniskamuseet.se/en/`
Address	Museivägen 7 PO box 27842, 115 93 Stockholm (Sweden)
Opening year	1938
Exhibition surface	10 000 m^2 indoor, 1 000 m^2 outdoor
N. of exhibits	6 math exhibits, plus 1 temporary
N. of visitors (yearly)	389 000

The Tekniska museum added an outdoors Mathematical Garden in 2018 with six installations, including various slides, mazes, fraction columns, musical instruments as xylophones and a dance mat, and many elements inspired by the golden ratio.

Chapter 3

Exhibits

The exhibits are ordered by the ordering of the museums (alphabetical) in the previous chapter.

3.1 Ammann rhombohedra

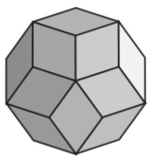
Deutsches Museum **Topic:** Polyhedra, tilings

Description

Puzzle. A set of 20 wooden pieces needs to be put together. There are two types of pieces (10 pieces of each type), but all of them are polyhedra consisting on six rhombic faces. All the faces of all the pieces are equal (a golden rhombus).

The goal is to combine them into a "ball" (rhombic triacontahedron). The label at Deutsches Museum reads:

> Instructions: The wooden Ammann Rhombohedra have to be put together into a ball. View from above:



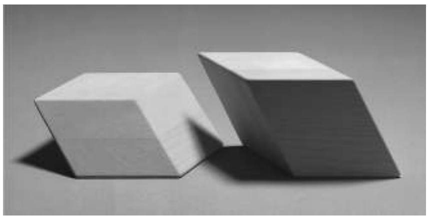
A polygonal cup with the appropriate dimensions helps supporting the construction.

Activities and user interaction

The typical user will understand quickly the goal and will spend a good amount of time trying to solve the riddle.

Many other constructions are possible using the golden rhombohedra (or Ammann's rhomohedra), so other challenges could be added. Furthermore, it would be a nice addition to have a pannel mentioning Ammann's aperiodical tilings, similar to Penrose ones.

Mathematical background



The faces of the pieces are golden rhombi, that is, a rhombus whose diagonals are in the golden ratio $\varphi = \frac{1+\sqrt{5}}{2}$. There are two types of pieces that one can construct from them: the *acute form*, in which there are two opposed vertices where three acute angles meet; and the *obtuse form*, where there are two opposed vertices where three obtuse angles meet.

These golden rhombohedra have rich combinatorial possibilities, for instance one can make the rhombic triacontahedron (ball-like) or the rhombic hexeconthaedron (star-like).

Besides the puzzle, it is very interesting its link with aperiodical tesellations. Penrose tilings of the plane are widely known, and in fact many math museums have an exhibit based on one of Penrose designs (although they usually lack an explanation of what *is* the Penrose aperiodic tiling). Less known are the Ammann tilings, and in particular, Ammann's aperiodic tiling of the three-dimensional space. Ammann tiling, is such that adding some "notches" or gluing rules to the faces of the two rhombohedra, one can obtain a space-filling tesellation of the space, which is non-periodic.

It would be a perfect companion to a plane Penrose tiling exhibit.

Resources

Wikipedia gives a good overview of polyhedral constructions. Also this article about Robert Ammann is worth reading in connection to this exhibit:

- Senechal, Marjorie (2004), *The Mysterious Mr. Ammann*, The Mathematical Intelligencer, 26 (4): 10–21, doi:10.1007/BF02985414, MR 2104463.

3.2 Polyhedron of triads

Erlebnisland Mathematik **Topic:** Music

Description

The exhibit is a column-like polyhedron consisting on triangular faces, as in a stack of five octahedra. The exhibit is 1.5m tall, on a wooden base which is 50cm tall. The edges and vertices are made of metal, the surfaces are equilateral triangles with edge length 35cm that are made of transparent plastic. If you touch a vertex, a musical tone plays. The name of the note is written onto the adjacent surfaces next to the corresponding vertex. The notes that belong to one surface always form a musical triad. The letters on the surface are small if it is a minor chord and capital if it is a major chord. There is a small sign that reads:

> The structure displays the geometry of harmonic intervals in diatonic music. The vertices correspond to the tones, the faces to the triads. Touching a vertex will cause the corresponding tone to sound. You can discover major and minor chords and the circle of fifths.

Activities and user interaction

Interaction is by touching and hearing. There is no supervision for this or any exhibit in particular but there are mediators around in the exhibition who you can ask questions.

Sometimes there are tours through the exhibition where visitors can explore the music-and-math exhibits, one of which is the polyhedron of triads. In this tour people can for example learn why triads and the twelve tone system make sense mathematically.

Children love to play with this exhibit but it is also still interesting when one knows a lot about music theory. As many people make music in their free time this is a good way to connect math with people's hobbies. The exhibit is very aesthetically pleasing.

Mathematical background

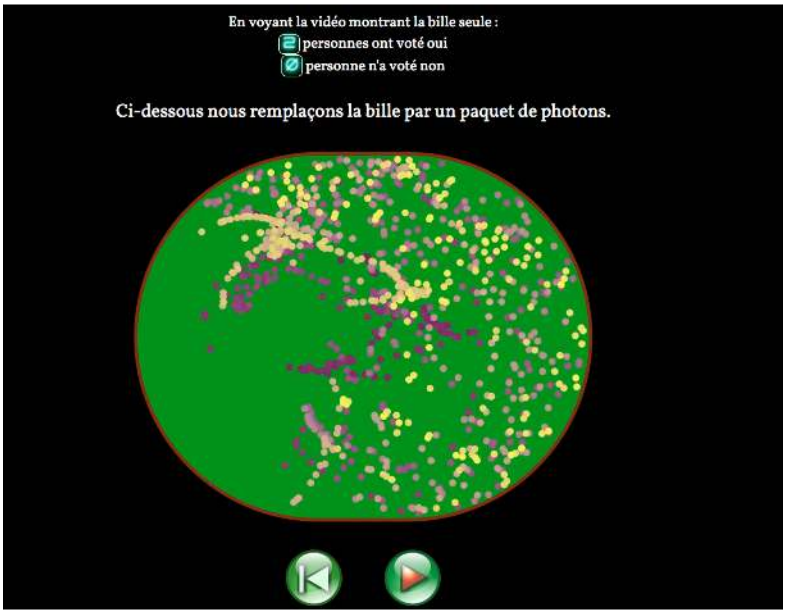
If we represent all minor and major triads by triangles, and connect their edges if they have two notes in common, we get a polyhedron which has the topology of a torus. We would get this polyhedron from the exhibit if we glued together the lowest and the upper row of triangles. In the geometry of the polyhedron we can discover all sorts of concepts from music theory: major chords belong to triangles that point upward, minor chords belong to triangles that point down, edges to the upper right belong to perfect fifths, edges to the upper left belong to minor thirds, horizontal edged to major thirds. Chords which are harmonically close, like parallel keys, are also close on the polyhedron. It is very interesting to analyze how typical progressions of chords used by composers look as paths on the polyhedron.

Resources

Conceived and designed by Bernhard Ganter.
Web: `http://www.erlebnisland-mathematik.de/en/portfolio/dreiklangpolyeder/`

3.3 Chaotic billiard

Fermat Science **Topic:** Chaos



Description

This virtual exhibit explores the understanding that we have about what is the meaning of "chaotic". Conceived as a quiz/survey, the visitor is shown a simulation of a ball bouncing endlessly on a billiard table with different shapes. For each shape, after seeing the animation, the user must decide if that behavior is chaotic or not. After the user's choice, the mathematician's answer is presented, which is not always "yes" "not" to be chaotic.

The exhibit is presented on a big touchscreen.

Activities and user interaction

The chaotic behavior is a concept difficult to approach. Every visitor has an idea that "chaotic" means disorder or unpredictable, but it is difficult to identify or measure.

The visitors are attracted by the fact that a billiard table is something real and tangible, not something abstract or big as the world's weather. Suddenly, understanding a complex mathematical concept is less scary for the visitor. Also, the fact that the exhibit is presented as a quiz, makes it an engaging challenge.

The main screen offers six shapes for a billiard table (stadium, lemmon, circle, rectangle, regular pentagon, and rectangle with circular obstacle inside). When we select one of the billiard tables, a first video is presented. On this video, a single ball is bouncing without friction for a few seconds. Then the user is prompted if that is chaotic in his opinion (options are yes, no, pass). Afterwards, a second video is presented, this time the ball is substituted by “photons”, or actually, by a bunch of balls that depart from the same point as before, but now their direction spread across a narrow angle (mathematically, a slight perturbation of the initial conditions). On this second video we can see all the balls bouncing at the same time, with their trajectories diverging slowly. After the second video, the question is repeated to confirm. After confirmation, the mathematical answer is presented. Sometimes the answer is “slightly chaotic”, or other nuances.

Mathematical background

Chaos is a concept in dynamical systems related to the high sensitivity to initial conditions. Even if the system is perfectly determinist (no random effects), the tiniest change in initial conditions (position, speed. . .) gets quicly amplified and the behavior differs enormously.

This is shown successfully in the exhibit by the “photons” simulation, since their initial vector speed only differs very slightly. But even with that mathematical concept, it is difficult to precise what does it mean to “differ enormously”, so at the end there is always a part of subjective appreciation, which makes more relevant to have the exhibit in quiz/survey form.

History and museology

The exhibit was conceived by Xavier Buff and Arnaud Chéritat for Fermat Science in 2016. It has been a succesful exhibit, with installations sold woldwide (USA, Singapore, French Antilles amongst others.)

Originally, the exhibit was part of a small exhibition about Poincaré, which made contributions to dynamical systems and chaos theory.

Resources

Website with the exhibit, and page on Fermat Science:

- `https://billard-chaotique.pagesperso-orange.fr/index.html`
- `https://www.voyage-mathematique.com/exposition/henri-poincar%C3%A9/`

3.4 Game of Tri

Haus der Mathematik **Topic:** Strategy

Description

On a wooden panel, six pegs (nails, screws...) are distributed on the vertices of a regular hexagon. A box of rubber bands of two colors is available. With the rubber bands we can join any two of the pegs forming a segment.

Activities and user interaction

It is a game for two players (each one has one color of the rubber bands). Alternatively, each player puts one band joining two of the pegs (that are not already joined). The goal is to be the first to make a triangle with three of the pegs as vertices (intersections of rubber bands are irrelevant).

The game immediately recalls Tic-tac-toe. However, this is much less known and most likely each visitor will spend some time trying to find a winning strategy. While on Tic-tac-toe there is a strategy to force a tie (so one can always avoid losing), that is not the case in the Game of Tri.

Mathematical background

The exhibit is based on the following article, that makes an analysis of the game and its pedagogical value with children:

- Haggard, Gary; Schonberger, Ann Koch (1977): *The Game of Tri*, Arithmetic Teacher, 24, 4, 218-320.

Resources

The game can be played online at the website of Haus der Mathematik.
`https://www.geogebra.org/m/yr2849ja`

3.5 Burning mirrors

Il Giardino di Archimede **Topic:** Conics

Description

The exhibit consists of a pair of parabolic mirrors, making a set of indoor "burning mirrors". Solar rays are substituted with those from a halogen lamp. The lamp is located in the focus of a first parabolic mirror, from which the light rays are reflected and come out parallel to the axis. A second reflection on the second mirror concentrate the light rays again on the focus of this second mirror.

Turning on the lamp, the heat generated by the bulb put in the focus of one mirror, manages to light a match in the focus of the other.

The parabolic mirrors can be found in the specialized market, the rest of the structure is usual workshop task.

Activities and user interaction

A mediator is always present to conduct the demonstration. The mediator puts a match in the focus of the second mirror, lights the lamp and the audience waits a few seconds for the match to be burnt. An explanation follows.

The lighting of the match makes the properties of paraboloids clear and visible: this fact stimulates the visitor's wonder and curiosity, beyond the strengthening of the concept.

Mathematical background

A tradition dating back to Plutarch says that Archimedes invented the mirrors with which set fire to the Roman ships besieging the city of Syracuse. How they built these mirrors Plutarch does not say.

The simplest shape of a burning mirror is that of a paraboloid of rotation, a surface that is generated by rotating a parabola around to its axis. In a parabola there exists a point F, called focus, with the properties that rays parallel to the axis concentrate in the focus.

This property of the parabola and paraboloids can be used to build a mirror that concentrates solar rays (which we may consider parallel because of the great distance of the Sun) in the focus, where they can light up inflammable materials.

History and museology

This exhibit is part of a series about conics sections in the main exhibition "Beyond the compass", at Giardino di Archimede. It is particularly connected with an exhibit on the paraboloid obtained from the rotation of a parabola and with an exhibit on the sound parabolas. Other exhibits concern the focal properties of the ellipse and the ellipsoid.

This exhibit was part of the seminal experience in 1991 by Prof. Franco Conti and Prof. Enrico Giusti, even before the Giardino was conceived.

Resources

- `http://php.math.unifi.it/archimede/archimede/curve/schede/schede_pdf/specchi.pdf`

3.6 Huygen's pendulum

Il Giardino di Archimede **Topic:** Physics, plane curves

Description

In the exhibit two pendulums are compared. The first one (left) is a free pendulum, moving along a circumference. The seocnd one (right) is a Huygen's, or cycloidal pendulum, where the weight moves along a cycloid. Two photocells measure their periods, that are visualised on a screen.

The two pendulums are as long as the ceiling allows, have the same length, and are attached to a wooden structure that supports all the contraption. Each pendulum has a double string, making a V shape, so the pendulum has two anchor points and oscillates in only one plane (as opposed to a single string pendulum that would move in all directions).

To make Huygen's pendulum, the string is attached on the top to a wooden curved template on each side. When the pendulum oscillates, the string is forced to rest in contact with the template, adopting the shape of a cycloid arch.

A sensor flips its state each time that the weight of the pendulum crosses the lowest point of its trajectory. The computer measures the state change and displays on the screen the duration of the oscillation period. The screens has two graphs, one per pendulum.

Activities and user interaction

As the pendulums start oscillating, we can see that while the period of the ordinary pendulum decreases with the amplitude of its oscillations, the one of the cycloidal pendulum is strictly constant. The screen is non-touch, the interface allows to record, pause and reset the graphs, although that would even be unnecessary by having the graph to reset itself after a few seconds. The main interaction of the visitor is not by touching the screen, but by releasing gently the weight of each pendulum at different heights.

In Giardino, this exhibit comes last after other exhibits introducing the cycloid's main properties (see Museology below). This exhibit helps to visualize an application of the cycloid through the Huygens' pendulum and to understand better its mathematical aspects, completing the properties exposition.

Mathematical background

Galileo had observed how the oscillations of a pendulum take more or less the same time, independently from the amplitude. Pendulum clocks use this principle but, in reality, the pendulum's oscillations are not exactly isochronous: the time it takes to complete an oscillation does depend on the oscillation's amplitude, and it is longer for wider oscillations. Only for very small oscillations (as in clocks) we can consider the time substantially constant.

How must a pendulum be so that the all its oscillations require strictly the same time, in Greek iso-chronous? More precisely, along which type of curve must a body oscillate so that the oscillations are perfectly isochronous? The answer is the cycloid.

But, how to make a pendulum oscillate along a cycloid instead of a circle? The trick is to use some profile so that the string rests in contact to it and describes what is called mathematically the *evolute* of the profile curve. It is a remarkable fact that the evolute of a cycloid is another cycloid equal to the first (this can be shown in another module). This is how the isochronous Huygen's pendulum is constructed.

History and museology

The exhibit is part of a series on the cycloid and its properties, included in the exhibition "Beyond the compass" from Il Giardino di Archimede. The other exhibits are:

1. Construction of a cycloid by rotating a wheel on a whiteboard,
2. Brachistochrone / isochrone property: by leaving a ball fall through a different tracks, we see that the cycloid minimizes falling time, and time is independent of the starting point.
3. Evolute property by a wooden profile on a whiteboard with a string.

Resources

`http://php.math.unifi.it/archimede/archimede/curve/schede/schede_pdf/cicloide1.pdf`

3.7 Clumsy Wagon

Matemateca **Topic:** Plane curves

Description

The exhibit presents a wooden board on a table. A groove (the 'rail') traced along a winding pathway goes from one side to the opposite side of the board. A piece of wood (the 'wagon') is placed over the groove with two pins inside it. The pins have 'heads' under the groove that prevent the wagon from being removed off the rail. As the wagon has always two of its points necessarily on the rail, its movement is particularly difficult if the path is very winding. The goal is to carry the wagon from one endpoint to the other.

Activities and user interaction

Users usually have no difficulty into figuring out what they have to do, after reading a hint or listening to the mediator instruction. Then they are surprised by how painful it is to handle the wagon in order to achieve the task. There are moreover some 'traps' in the way, where the user may wander in circles if they do not stop to think at some special positions.

The exhibit poses a very unusual problem, but in some sense also very real. It is defying to see a unidimensional path without bifurcations making people lost. The path does not indicate clearly where are the difficulties and what will be the necessary movement to cross it from one side to the other. This is so because the 'right' way of seeing the problem is through the map of level curves of a two-dimensional function. In the end, they get there, but a mysterious atmosphere remains.

Mathematical background

This exhibit is inspired by the "Moving needle problem", appeared in the blog Area 777, authored by Conan Wu.

There is a conjecture saying that wagons of arbitrary size can cross arbitrarily chosen paths if they start and end at two points of the same line. The conjecture is stated for C^r smooth paths, with r greater than zero, since there are C^0 counter-examples.

The interesting point here is that the problem can be reformulated in terms of the connectivity of level curves of a suitable two-variable function.

History and museology

The exhibit originated in Matemateca, inspired by the 'moving needle problem' in the mentioned blog, Area 777. We are not aware that any other museum constructed something similar so far.

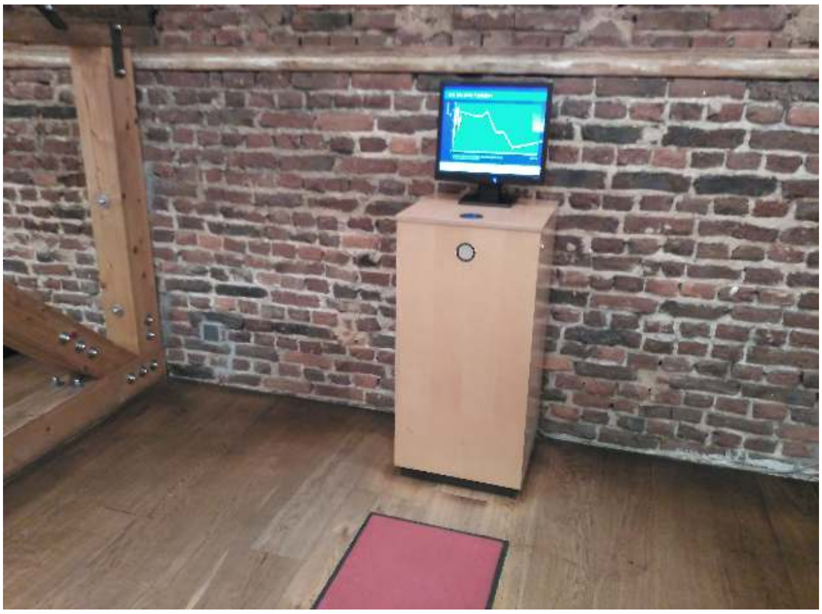
Resources

Blog post in Area 777:

- `https://conan777.wordpress.com/2010/11/22/the-moving-needle-problem/`

3.8 I am a function

Mathematikum **Topic:** Functions



Description

A screen is presented on top of a wooden stand, with a long carpet in front of it. As we walk on the carpet towards the screen, a sensor measures the distance between the stand and us. The screen shows a fixed (white) graph of a function, and as we move on the carpet a new (yellow) graph of our distance vs time is drawn. The aim is to come the given graph as close as possible. The carpet has markings showing the distance to the stand. A button next to the screen allows to reset the exhibit and a new white function graph appears.

Activities and user interaction

The screen is non-touch, and it becomes obvious that the only button is a reset. The users immediately realize that the yellow graph is related to their position by means of some sensor.

The height of the curve is the distance to the stand, which can be compared with the carpet markings. Moving forwards towards the stand makes the function to decrease, while moving backwards makes the graph to rise. All didactical aspects of a function can be experienced: the values at a certain point (where do I have to start?), the variational aspect (do I have to walk slow or fast, and in which direction?), the function as a whole (could I repeat my walk without looking on the screen?)

The fact that all these mathematical aspects are experienced but not verbalized (minima, maxima, slope, etc.) keeps the activity fun and playful, while the visitor is conscious of the mathematical ideas. Visitors of all ages do it with pleasure, others can observe it.

Additional activities may be doing that looking away from the screen, with the help of a friend that gives instructions, or trying to trick the machine to draw a discontinuity.

Mathematical background

All aspects of a function (slope, extrema etc.) may be discussed/experienced on this exhibit.

History and museology

The idea of this exhibit can be traced back to the Texas Instruments calculator-based ranger (CBR), a sonic motion detector sold for TI-82 and other calculators with the package "Ranger" around 1997. There are many copies and versions of this exhibit in the world.

Resources

Cf. with the catalog(s) of Mathematikum,

- *Learning Math with Interactive Experiments. 45 Experiments from Mathematikum.* Giessen, 2016.
- Beutelspacher, A., *Wie man in eine Seifenblase schlüpft*, C. H. Beck, Munich, 2015. ISBN 978 3 406 68135 6. (in German)

3.9 Parabola bounces

MathsWorldUK **Topic:** Conics, plane curves

Description

The exhibit consists on a metal band bent in the shape of a parabola (with big aperture), and placed between two large sheets of transparent acrylic plastic walls. All is supported by a wooden structure with the axis of the parabola vertical from the floor. The height of the exhibit is around 1m. Users can drop a ball to fall between the two transparent walls. The ball then bounces against the metal parabolic band and hits a bell which is located in the focus of the parabola. The ball eventually reaches the bottom of the parabola and exits the wall through a hole.

The height of the bell can be adjusted, and some tubes help dropping the balls perfectly vertical.

Activities and user interaction

Visitors can try to drop the ball from different distances to the axis, or from different heights.

There are practical problems, because with the action being vertical gravity interferes (thus producing more parabolas). However, this effect is small to be detected in most of the cases with the dimensions of this exhibit. In fact, this sparks discussion amongst adult visitors and even engineers and professors enjoy arguing about the relative effect of different factors.

The exhibit is perceived as a challenge, and it makes a loud noise when the bell is hit, which makes the experience much more attractive.

Mathematical background

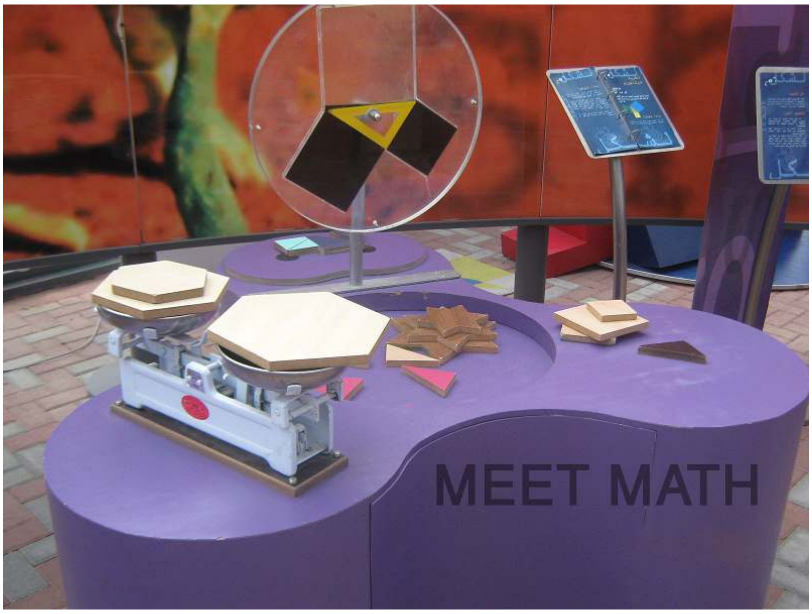
Reflective properties of parabolas are classic. A panel with a description of the parabola and some of its properties is next to the exhibit.

History and museology

This exhibit of MathsWoldUK is a prototype for their traveling exhibition, but it has proven to be an attractor of the interest of all visitors.

3.10 Pythagoras' Theorem (set)

Meet Math Museum, MMACA, and many others **Topic:** Pythagorean theorem



Liquid disk, Pythagorean balance and decompostion puzzles
(Meet Math Museum)

Description

On this exhibit we include many modules around the Pythagorean theorem.

Liquid disk. An acrylic disk that can be rotated on a support. In its interior, a right triangle and three squares over its sides. The squares are empty, and a liquid can flow between the three squares. By rotating the disk, the liquid that fills the two smaller squares can be made to fill the bigger one.

Decomposition puzzles. Puzzles usually made of wooden pieces. A right triangle is glued on the table. With the wooden pieces, two squares can be constructed over the legs of the triangle, or re-arranged to form a single square over the hypothenuse. Sometimes the shapes to construct over the sides are not squares, but other shapes polygons, giving an additional nice touch.

Pythagorean balance. Several pieces of different shapes (hexagons, half-circles...) in sets of three, all the three in each set with the same shape but different size, that fit to the size of the sides of a fixed right triangle. A balance (usually of type Roberval) allows to compare weights, to verify that no matter the shape, the sum of the weights of the two smaller is equal to the weight of the bigger.

Other jigsaw constructions. We include here the pythagorean window, the pythagorean rings, lunulae constructions, and many others that can be found in museums.

Activities and user interaction

This super-classical subject is still a big success in math museums. This is due to the familiarity of the public with the theorem (but not with these physical realizations), the fact that it is closely related to the school curriculum (teachers demand it), the feeling of understanding, and for the adult public the pleasure of being surprised by something supposedly so exhausted.

Few instructions are necessary to induce users to handle the exhibits. However, it is important to make some remarks (by panels or by mediators):

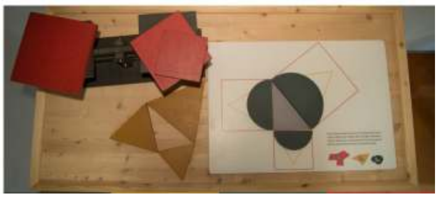
- Underline the relationship between weight and areas (due to the fact that all shapes have the same thickness).
- That something that looks true is not always true. Compare with some apparent paradoxes where some misalignments induce to see vanishing areas.
- Most importantly, remark that a verification is not a proof. The proof must be an argument. In particular, why does that particular fact remains true for any other right triangle? Why the argument does not depend on this physical realization?

Mathematical background

There are over 400 known proofs of the Pythagorean theorem. Many of them are susceptible to be adapted to a museum exhibit.

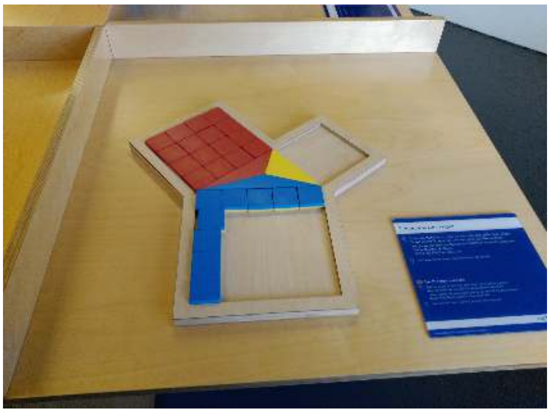
History and museology

Almost every math museum has some exhibit related to the Pythagorean theorem, some of the reasons are pointed above. Some exhibits, like the liquid disk or the Pythagorean balance, are widely spread. Others are much less known and still surprise the most experienced visitors. We are aware of those of Mathematikum; MMACA, which makes special teacher training around that topic; and Il Giardino di Archimede, that created an exhibition named "Phythagoras and his theorem".



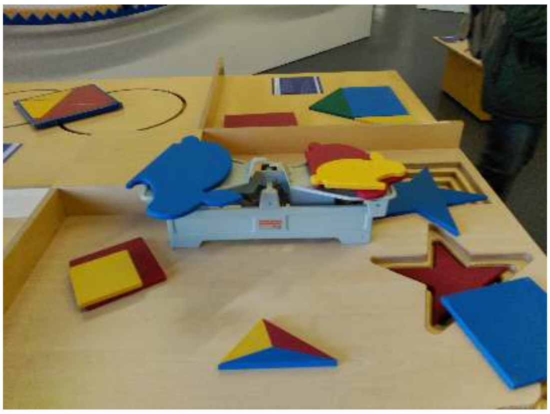
Pythagorean balance (MMACA)

Decomposition puzzle with hinge (Mathematikum)

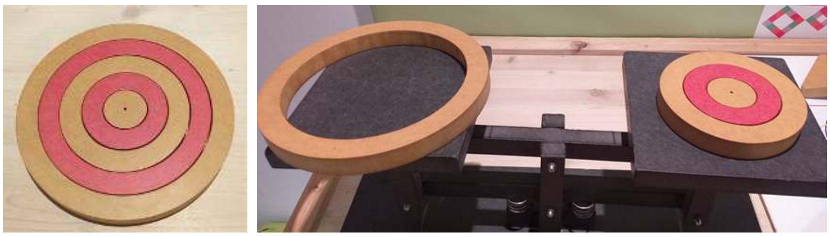


Decomposition on a 3-4-5 triangle (Mathematikum)

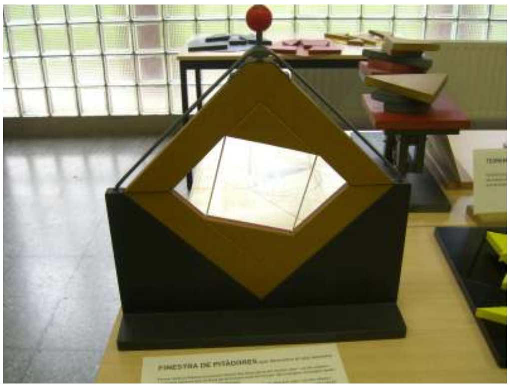
Pythagorean balance (Experiminta)



Pythagorean balance (Mathematikum)



Pythagorean rings (MMACA)

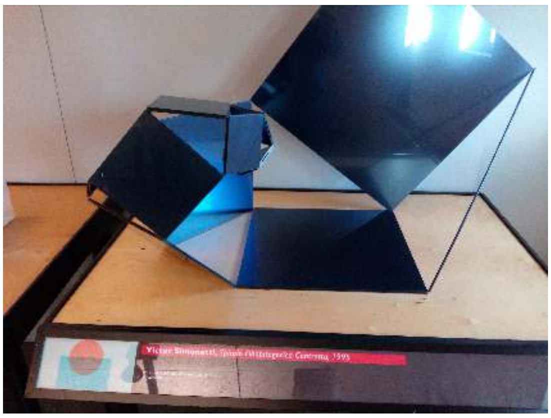


Pythagorean window (MMACA)

Decomposition puzzle (Il Giardino di Archimede)

Decomposition puzzle with hexagons (Il Giardino di Archimede)

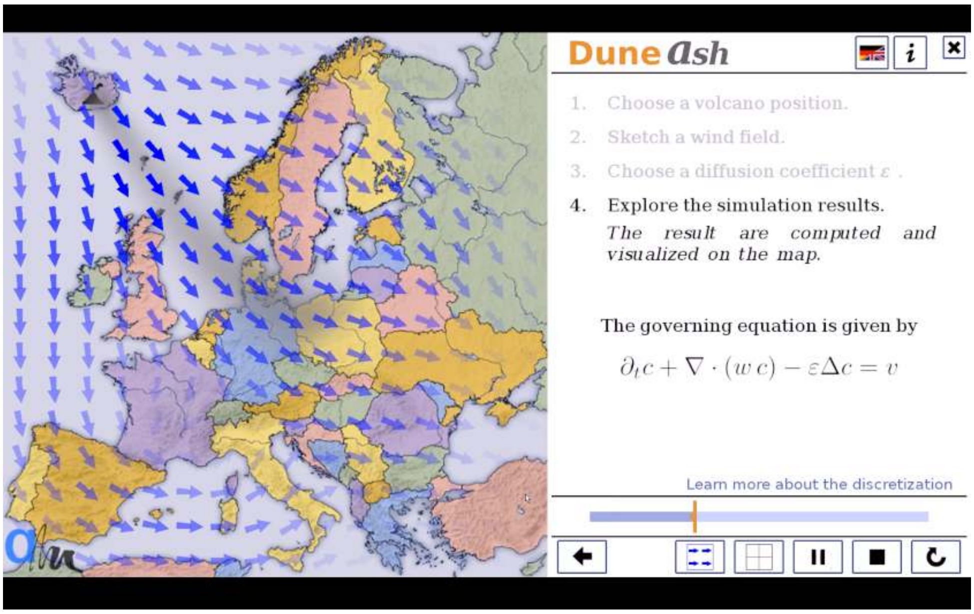
Pythagorean Lunula (Il Giardino di Archimede)



Artistic sculpture using the Pythagorean theorem (Il Giardino di Archimede)

3.11 Dune Ash

MiMa **Topic:** Modelisation, fluid dynamics



Description

This virtual exhibit is an interactive simulation of a volcano eruption in Europe. You can place a volcano, add a wind field and explore the ash cloud dispersing in time.

The program simulates how an ash cloud emitted by a volcano moves in a certain wind field which the user can draw on a map of Europe. The wind, that is, the velocity, does not change in time, as it would happen in reality. Like that it is clear and easy to see what happens when the volcano erupts and the ash cloud is moving over Europe. The ash transport is simulated by solving a partial differential equation (PDE).

The exhibit is installed on a big size touchscreen.

Activities and user interaction

The screen is divided in a left pane with the map, and a right pane with the instructions and controls. There is a restart button, or otherwise the restart is triggered after a period of inactivity.

The right panel has always the whole user instructions as an enumerated list.

1. Choose a volcano position.
2. Sketch a wind field.
3. Choose a diffusion coefficient ϵ.

4. Explore the simulation results.

The active step is highlighted, and the others are grey. On each step, a few controls appear or disappear from the right panel. We can move from one step to another with a "back" and "next" buttons.

On Step 1, we can drag an icon on the map to choose the position. On Step 2, we can move our finger over the map to draw some lines representing the wind, the faster we move, the stronger is the wind. After this step, a computation triggers to have a wind field, that is, a vector field on each point that approximately resembles the lines introduced by hand. On Step 3 we can choose the diffusion coefficient ϵ, which accounts for micro-turbulences. On screen appears the partial differential equation with the parameter ϵ, and a brief explanation of the significance of each term. After this step, the main computation triggers, computing a source of ash particles at the volcano, and integrating their trajectories across the wind field according to the diffusion equation. The computation yields a collection of frames or snapshots, that we can play as a movie, or move back and forward in time with a slider.

The exhibit is self-explanatory and does not require a mediator, although if present, he/she can help stressing the applications of mathematics to understanding the Earth phenomena. This exhibit includes some research-level mathematics on the backstage, and it is not the intention to explain the equation or the numerical methods used to solve it to the public, but to show current, useful uses of mathematics and to display some of what mathematicians do today.

Mathematical background

A more detailed documentation is provided by the authors, see the Resources section.

History and museology

The exhibit was developed for a "Science market" by the research team of Prof. D. Kröner at Alber-Ludwigs-Universität Freiburg. The background are the eruptions of the Icelandic volcanoes Eyjafjallajökull (2010) and Grímsvötn (2011), that blocked the aerial traffic for days or weeks across Europe.

The program was presented to the math exhibits competition Mathematics of Planet Earth 2013, called by IMU and UNESCO, and organized by IMAGINARY. It was awarded the second prize. Since 2013 it is part of many regular exhibitions of IMAGINARY, and there are some permanent installations in museums, like MiMa, Deutsches Museum, Experiminta, and others.

Resources

The exhibit is released under free/open source licenses (GPL 3.0 and CC BY-NC-SA 3.0). All the content (program and documentation) is available on the website:

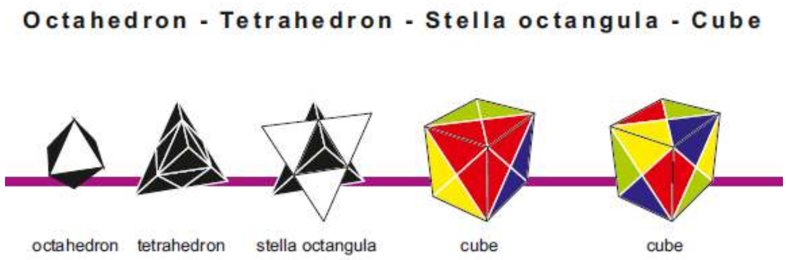
- `https://imaginary.org/program/dune-ash`

3.12 From Octahedron to cube

MMACA **Topic:** Polyhedra

Description

The exhibit is a puzzle of 21 wooden pieces linkable by magnets. The core piece is a black and white octahedron. Its diagonal has the same length as the edge of the cube that represents the solution of the puzzle. This octahedron is the dual polyhedron of the cube. The other pieces are 8 regular and 12 irregular tetrahedrons.



Constructive sequence from an octahedron to a cube.

Activities and user interaction

The activity proposed in the exhibition is simply to build the reproduced solids in the illustrated sequence, starting with the regular octahedron in the centre of the structure.

An added difficulty is to correctly combine the pieces so that the faces of the cube always present the four colours.

It is a fairly easy activity that can play any type of public in a time appropriate to the rhythms of an exposure. The magnets hidden in the faces of the different solids make the activity quick and pleasant.

A mediator can propose some activities that allow for an "instant workshop", or altenatively for a use in the classroom. See the didactical guide in the Resources section below.

Resources

Didactical guide and catalog from MMACA:

- `https://www.mathcom.wiki/index.php?title=File:Octa2cube_activities.pdf`
- Museu de Matemàtiques de Catalunya. *Catàleg*. ISBN 978-1-36-434607-2.

3.13 Polyhedral kaleidoscopes

MMACA **Topic:** Polyhedra, mirrors, symmetry

Description

The exhibit consists in a set of three spherical kaleidoscopes. Each kaleidoscope is made of four equal mirrors in the shape of circle sectors arranged on an inverted pyramid structure. A set of colored wooden pieces accompanies each kaleidoscope, so that these pieces fit inside the mirrored pyramid. As a result of the reflected piece, one sees several kinds of symmetric polyhedra. Each kaleidoscope has a different set of symmetries, depending on the dihedral angles of the mirrors. Those are $(120, 120, 120, 120)$, $(120, 90, 120, 90)$ and $(120, 72, 120, 72)$ degrees, giving locally at each edge dihedral symmetries of order $(3, 3, 3, 3)$, $(3, 4, 3, 4)$ and $(3, 5, 3, 5)$ respectively. The four mirrors on each kaleidoscope are equal, and they span a circular sector of angle $70.53°$, $54.74°$, and $37.38°$.

Materials: Mirrors are glass-made, glued over plywood. Colored pieces made of MDF wood with tinted wax.

Activities and user interaction

The activities adapt to the age and background of the visitor. For younger visitors (5-10 years old), it is sufficient to bring them the wooden pieces and let them discover the beautiful shapes that are obtained. Next, play to identify the shapes with the list of polyhedra on the wall. For an intermediate level, one can challenge them to tell in advance which of the polyhedra on the poster will appear when putting a wooden piece inside the kaleidoscope, and induce them to see the number of copies seen at each corner.

This number of copies around the edge of the kaleidoscope depends on the angle of the two mirrors meeting at this edge. Discuss why the three different kaleidoscopes give rise to three families of polyhedra. For advanced visitors, one can discuss the notion of group and symmetries. Each family of polyhedra share a symmetry group, which is generated by the reflections on the mirrors. This way, one see that the abstract concept of symmetry is related to the structure that provides the kaleidoscope, but the actual shape depends on the particular piece that we put inside.

Mathematical background

Depending on the definition we choose, we can make lists of hundreds of polyhedra, grouped by different characteristics. Here we see that more important than the number of faces, or their shape, a much more significant feature is which symmetries does it has. All the polyhedra that fit on one of the kaleidoscopes share the same symmetries, which are different from those of another kaleidoscope. A symmetry in this context is a movement that one can do and leaves the polyhedron unchanged (for instance rotating a cube a quarter of turn around the line that crosses two opposite faces by their centers). For polyhedra, symmetries can be rotations or reflections. A polyhedron can be embedded into a concentric sphere, and all the symmetries of the polyhedron are movements of the sphere. The angles of the mirror circular sectors relate to the dihedral angles by spherical trigonometry. Similar relations are needed to calculate the bevels on each face (wooden pieces). The set of symmetries of a polyhedron has the mathematical structure of a group. Groups can be studied abstractly, but here we find a perfect application of group theory to geometry. One can mention that all these groups of kaleidoscopes are generated by order 2 elements, and hence are Coxeter groups, which are completely classified.

History and museology

This exhibit has become an iconic feature of MMACA, and it is usually part of MMACA's travelling exhibitions. Although there are precedents (Matemilano exhibition, Milano, Italy, 2003-04), this exhibit does not use the minimal set of three mirrors to generate the polyhedra, but sets of four mirrors and symmetric wooden pieces. This has the advantage of being stable (the pieces hold in place) and the aperture of the basket is bigger, trapping light and offering a good view of the polyhedra. It is believed to be the first exhibit with this four mirrors design. Similar exhibits can be found at Erlebnisland mathematik (Germany), Le Labosaïque (France), and Atractor (Portugal).

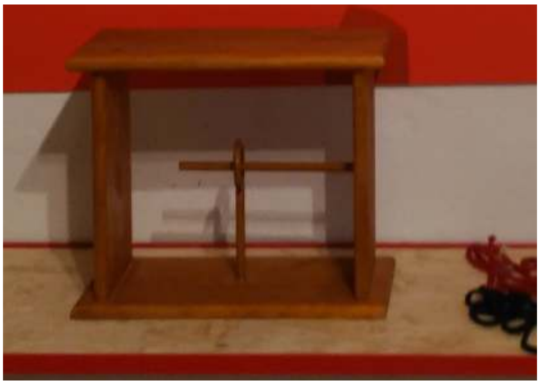
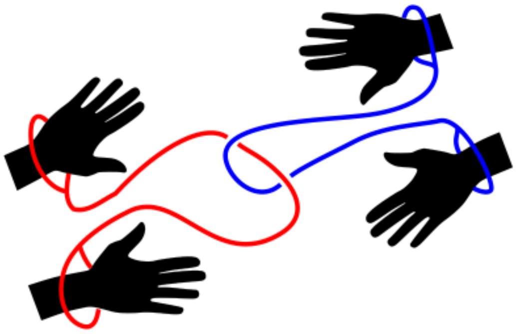
Resources

The following books are a good source for constructing kaleidoscopes:

- M. Wenninger, *Spherical models.*
- H.S.M Coxeter, *Regular polytopes.*

3.14 Knots and links

Museo de Matemáticas de Casbas **Topic:** Topology, knots

Description

The game only needs two ropes for the people to interact. Additionally, a wooden model serves as hint and explanation.

To set up the game, the mediator picks two visitors, and ties them according to the figure: each visitor has both hands tied together with a rope that surrounds each wrist. The loop around the wrist closes with a tight knot, but leaves some space between the wrist and the rope, although not enough to slip the hand off. The two visitors are attached together because their chest, arms, and rope make two linked rings. The goal is to unlink themselves without untying the knots around their wrists.

The wooden model is a frame (such as a box without front and back walls), from the bottom side a stick goes towards the center, and holds a ring. This stick with ring has the shape of an eye bolt. A second stick goes from a side wall towards the center, and penetrates without touching the ring sustained by the first stick. The first stick represents the rope with the loop around the wrist, and the second stick represents the arm.

After playing around for a bit, the mediator uses the wooden model as a hint: it explains what the sticks represents, and puts another rope on the model, as it is on the real situation. It becomes apparent that the solution comes by using the space between the loop and the wrist.

Activities and user interaction

The mediator sets up the situation and the two visitors need to think together. It works great with children or adults, and the situation is likely to become a tangle and laughs are assured.

The wooden model really helps to the reflection of what does it mean to be linked; mathematically, the two visitors are never linked. It also helps to find the solution by reflection and not by blindly trying to move things around.

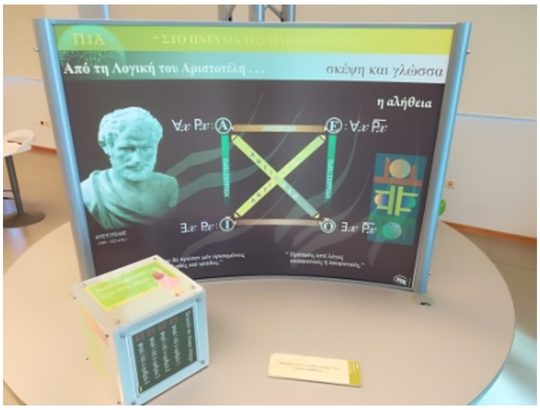
Mathematical background

Topology remains a big unknown domain for most of the general public, which will ask why that is mathematics. Topology is a branch of mathematics that deals with deformation of objects, without attending to measurement of any distance. Knots are such objects: it is not important how long are the ropes, or if they are straight or curved, just matters the connectivity between the links.

But without telling the visitors that there is a branch of mathematics to study knots, just the fact that one has to make reasonings, and define precisely what does it mean to be linked, makes them to see that it is related to mathematics.

3.15 Aristotle's Logic

Patras Science Center **Topic:** Logic



Description

This exhibit introduces the basic principles of logical and mathematical thinking.

The module consists in two cubes made of plexiglass, containing enclosed a piece of paper on each face.

On the first cube, there is one statement on each of the six faces. Four of these sentences make reference to the others, and the task is to decide which of these propositions are true and which false. The other two sentences have a gap, that must be filled in to make that proposition true.

On the second cube, there are various independent syllogisms and enigmas and the task is to reach a conclusion on each one.

Next to the cubes there is additional instructions to motivate the puzzle.

Activities and user interaction

The cubes are designed to be used by visitors of any age from 6-7 years old.

For the younger ones the best activity is to fill-in the missing part of the propositions of the first cube, and try to reach a conclusion of a simple enigma or a simple syllogism on the second one.

The older ones (14 years and up) can deal with all six faces of the cube, and can also explain why the conclusion reached in some simple syllogisms appears to be bizarre.

For a more advanced activity the participant is asked to use symbols in order to code the syllogism and reach a conclusion faster. This is achieved on workshops of 1.5h offered at Patras Science Center.

Mathematical background

The fist cube deals with the notion of logical proposition. In order for a simple phrase to become a logical proposition a criterion of truth is needed.

Two or more logical propositions can be related through conjunction and disjunction. These two logical operations are core elements of set theory identified as intersection and union respectively.

The related poster refers to the "square of opposition" where affirmation, negation, universal and particular can lead to all kinds of logical propositions. The difference between contradiction and contrariety can be easily understood: two contradictory propositions have both the same universal subject but different predicates (one affirmative, one negative) and two contrary propositions differ both in subject (one universal, one particular) and predicate (one affirmative, one negative).

The second cube introduces the notion of syllogism and the conditions (validity and soundness) under which a syllogism becomes a proof.

History and museology

In Patras Science Center, this module is part of the exhibition on mathematics and informatics. The module is surrounded by information posters on greek philosophers, the foundations of mathematics in logic, and references to formal logic.

As far as we know, there are no similar exhibits in museums we have already visited.

Resources

`https://patrassciencecentre.wordpress.com/infexhib/`

3.16 Platonic solids

Ramanujan Math Park **Topic:** Polyhedra

Description

The five platonic solids are built as painted steel frames in big size, and anchored to the ground in concrete pedestals. The sculptures are big enough as to allow children to get inside. There are some eye bolts attached to the edges of the polyhedron, that allow a rope to be anchored to specific points of the edge (middle points or other points with geometric properties).

Activities and user interaction

Many activities can be done exploring the polyhedra, from counting faces, edges, and vertices, to verify Euler's formula, to measure angles (face and dihedral) or lengths of segments, to drawing the polyhedra to get familiarity, or to see the projection by the Sun on the floor.

A special activity is conducted with a rope. The rope can be attached and passed through the eye bolts thus creating new edges of new polyhedra, inscribed on the steel polyhedron. Different degrees of complexity are possible. For instance, a first activity would be joining the middle points of each edge. Another would be to to attach a piece of a different string to the previous rope and make nested polyhedra.

Mathematical background

If we join the midpoints of the edges of a tetrahedron, we obtain an octahedron, which is easy to see and identify. If we join the midpoints of edges of an octahedron or a cube, we obtain a cuboctahedron, and if we do the same process with an icosahedron or a dodecahedron, we obtain an icosidodecahedron. The names or even the shapes are not so interesting as the fact that we obtain the same result for two pairs. This comes from the fact that these are pairs of dual polyhedra (having one the vertices at the center of the faces of the other), but they have the same number of edges and at the same positions, just rotated 90°. This can be seen in the giant models or with a smaller one.

Other constructions less obvious but very pleasant are for instance that if we join the edges of an octahedron not by the midpoints, but by the ratio $\frac{1}{\varphi}$, where $\varphi = \frac{1+\sqrt{5}}{2}$ is the golden ratio, then we obtain a perfectly regular icosahedron.

These and many other properties can be explored while developing a spatial vision, getting a familiarity with the objects, and intuition on their relations.

History and museology

The exhibits were conceived by V.S. Sastry, a Math Communicartor, and were executed along with Sujatha Ramdorai, a Mathematics Researcher.

Resources

A video of the Ramanujan Math Park

- `https://youtu.be/UdkrrcT9_lM`

A couple of books about polyhedra from a visual perspective:

- Pugh, A., *Polyhedra: a visual approach*, University of California Press, 1976.
- Wenninger, M. J., *Polyhedron models*, Cambridge University Press, 1974.

3.17 Nautilus Slide

Tekniska museet **Topic:** Plane curves

Description

A slide of steel in the shape of a nautilus shell is the central piece in Tekniska's Mathematical Garden. This is true figuratively (it attracts attention); and literally, since the garden has the shape of a giant spiral and the slide is in the center.

Activities and user interaction

The slide has a instruction sign and there is also information in different levels about the mathematical aspects of the Nautilus shell and the connections to both Fibonacci and the golden ratio and spiral.

The purpose is to experience math with your body and to encourage curiosity for math. And visitors love it!

Mathematical background

The nautilus shell is one of the classics for describing connections between math, art and nature. It is said to be connected to Fibonacci and the golden ratio. Nautilus shells (as other spirals in nature such as galaxies), follow logarithmic spirals, which have the polar equation $r = e^{k\theta}$. That is because logarithmic spirals appear in systems or organisms with constant growth. For a certain value of k, a logarithmic spiral is called golden spiral, and it can be approximated with great accuracy by arcs of quarter of a circle, in nested squares, tiling a golden ratio rectangle. This is depicted in the Mathematical Garden. Nautilus do not follow exactly that circle-piecewise spiral, real nautilus follow a logarithmic spiral and have many different growth rates (different values of k). However, it is undeniable that the spiral and the Nautilus Slide have a big amount of mathematics, art and beauty.

History and museology

The idea of making such a slide was a result of a workshop with artists, mathematicians, teachers and architects. The slide is completely custom made.

Aerial view of Tekniska's Mathematical Garden.